On researching thermoelastic instability.

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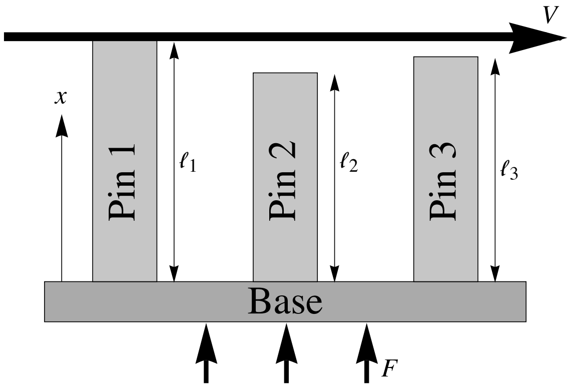
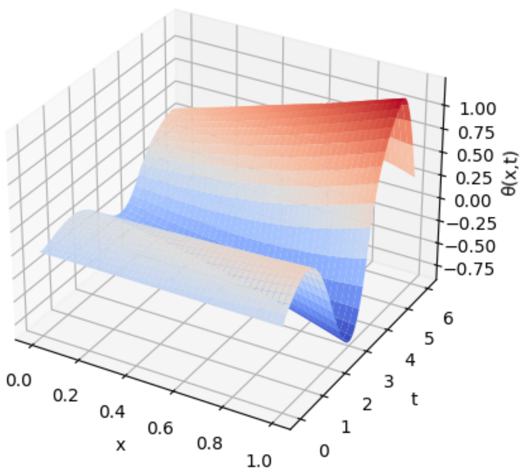
For my Rokos Award Internship, I worked with Prof. Peter Howell to develop and research new mathematical models for thermoelastic instability in physical systems.

In 1969, Prof. Jim Barber investigated the effects that wear and thermal expansion have on a system of metal pins which experience friction due to a moving surface [1]. In 2016, Barber worked with Prof. John Ockendon to develop this initial model further in order to account for physical effects exhibited in systems such as brakes and clutches, for example *hot spotting* and *judder* [2].
In 2018, Ockendon worked together with Prof. Peter Howell to explore further details of the mathematical model, developing the theory of oscillatory behaviour that is exhibited by the system [3] . In 2022, I worked with Howell to improve the model by introducing more realistic physical effects.



I began the project by getting to grips with the existing work on the problem and familiarising myself with computational mathematics, which I would use extensively throughout the project to simulate physical systems. I was able to use the experience and techniques that I learnt from undergraduate courses such as *Fourier Series & PDEs*, *Differential Equations 2*, and *Complex Analysis*. I used Python and the NumPy and SciPi libraries to implement techniques for numerically solving differential equations that model the temperature in the metal pins, and how this changes over time due to the physical effects of friction on the pins.



In my regular meetings at the Maths Institute with Prof. Howell, we discussed what new features we could introduce to the model to more accurately capture the true behaviour of the physical system. The inclusion of pressure-dependent contact resistance on temperature models is of interest to analysis of physical systems, and so we sought to include this feature in the model. This means that the rate of change of temperature at the end of the pin depends not only on the force applied, but also on the temperature at the end of the pin.

The next task was to investigate how the inclusion of pressure-dependent contact resistance changes the stability of the system for prescribed physical parameters which determine the rate of thermal expansion and wear. To do this, I used the symbolic manipulation software Mathematica. As the inclusion of this new effect significantly complicates the expressions for the "stability boundary", it was harder to analyse the qualitative behaviour of the boundary, although it emerged that for very specific parameter values of all three physical effects: expansion, wear, and rate of heat transfer, the new stability boundary separates the parameter space into more than two regions, as was observed previously. It was this discovery that allowed Peter Howell to develop the theory of new oscillatory behaviour in the 2-pin case - specifically periodic solutions with both pins in contact for all time, which I verified numerically using numerical simulations.

The Rokos Award Internship allowed me to stay in Oxford for the duration of my project, and facilitated regular meetings with Prof Peter Howell. I was also able to talk with Prof. John Ockendon about the problem, and he provided insight and guidance on what possible areas would be useful to explore. Me and my supervisor are planning to publish our work in the Journal of the Mechanics and Physics of Solids

References

[1] J. R. Barber. Thermoelastic instabilities in the sliding of comforming solids. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, 312(1510):381–394, 1969.

[2] J. R. Ockendon and J. R. Barber. A model for thermoelastic contact oscillations. *IMA J. Appl. Math.*, 81(4):679–687, 2016.

[3] P. D. Howell, J. R. Barber, and J. R. Ockendon. Multiple-contact thermoelastic oscillations. *J. Thermal Stresses*, 41(10-12):1329–1345, 2018.